The solution of equation (27) can be found by separation of variables. The function

$$w_h(r,t) = R(r)\ T(t) \tag{28}$$

is a solution, provided

$$\nu\left(R''T + \frac{1}{r}R'T\right) - RT' = 0 \tag{29}$$

or

$$\frac{T'}{\nu T} = \frac{1}{R}\left(R'' + \frac{1}{r}R'\right) \tag{30}$$

Since the member on the left is a function of time alone and that on the right is a function of the radius alone, they must be equal to a constant, say $-\lambda^2$; hence we have the equations

$$rR'' + R' + \lambda^2 rR = 0 \tag{31}$$

$$T' + \lambda^2 \nu T = 0 \tag{32}$$

Equation (31) is Bessel's equation, and its general solution is written as

$$R(r) = A\ J_0(\lambda r) + B\ Y_0(\lambda r) \tag{33}$$

where $J_0(\lambda r)$ and $Y_0(\lambda r)$ are Bessel functions of the first and second kind, respectively. G. N. Watson in reference (d) states that $Y_0(\lambda r)$ is infinite for interior problems, and consequently, $B = 0$. Thus the solution of equation (31) becomes

$$R(r) = A\ J_0(\lambda r) \tag{34}$$

The solution of equation (32), when ν is a constant, is

$$T(t) = De^{-\lambda^2 \nu t} \tag{35}$$

Then, upon substitution of (34) and (35) into (28), we immediately have

$$w_h(r,t) = C\, J_0(\lambda r) e^{-\lambda^2 \nu t} \tag{36}$$

A series of these solutions

$$w_h(r,t) = \sum_{j=1}^{\infty} C_j J_0(\lambda_j r) e^{-\lambda_j^2 \nu t} \tag{37}$$

represents a particular solution of the homogeneous equation (27).

A particular solution of equation (23) is

$$w_p(r,t) = -\frac{P_g \theta_1}{\rho L} e^{-t/\theta_1} \tag{38}$$

The sum of equations (37) and (38) represents the solution of the governing equation of the fluid velocity within the bore of the knock-off tube. Thus

$$w(r,t) = \sum_{j=1}^{\infty} C_j J_0(\lambda_j r) e^{-\lambda_j^2 \nu t} - \frac{P_g \theta_1}{\rho L} e^{-t/\theta_1} \tag{39}$$

In order to find the solution of equation (23) that satisfies the boundary conditions, we are motivated to reconstruct (39) in the form

$$w(r,t) = \sum_{j=1}^{\infty} \frac{C_j}{A_j} J_0(\lambda_j r) \left[e^{-\lambda_j^2 \nu t} - e^{-t/\theta_1} \right] \tag{40}$$